



PRINCETON UNIVERSITY

DEPARTMENT OF AEROSPACE AND MECHANICAL SCIENCES

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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Launch Vehicle and Propulsion Programs Division
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AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH

Status Report for the Period

1 April through 31 December 1966

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Aerospace Systems and Mission Analysis Research (ASMAR) Program
Department of Aerospace and Mechanical Sciences
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PRINCETON UNIVERSITY

AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH Status Report for the Period 1 April through 31 December 1966

ABSTRACT

After three years the work of the ASMAR Program at Princeton began to show maturity especially in the area of spaceflight trajectory analysis research. At the request of NASA Headquarters assistance was, and continues to be, provided the Jet Propulsion Laboratory in analyzing the performance of solar electric propulsion for solar system exploration. These mission studies are being undertaken under separate NASA Contract No. NSR 31-001-078.

Theoretical work continues and is being reported in the spaceflight trajectory analysis topic area on the primer vector and optimal impulsive trajectories in heliocentric flight. Work is also progressing on optimal planetocentric maneuvers and on the optimum patching with the heliocentric trajectories.

Efforts continue to identify methods for aerospace systems analysis. Work is underway on nuclear rocket and nuclear-electric power systems.

Although practically all of our efforts have gone into studies of solar electric missions, the computer programs being developed will be designed to analyze nuclear-electric powered and high thrust missions as well.

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AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH * Status Report for the Period 1 April through 31 December 1966

I. INTRODUCTION

A. General

The overall effort of the Aerospace Systems and Mission Analysis Research (ASMAR) Program in the Department of Aerospace and Mechanical Sciences at Princeton showed a developing maturity, especially in the area of spaceflight trajectory analysis, during the period 1 April through 31 December 1966. Work on the Basic Program under this contract was continued in all topic areas, although at a somewhat reduced pace for a variety of reasons as discussed in the following two paragraphs.

At the request of Mr. J. Mullin of the NASA-OART Electric

Propulsion Section, it was agreed to develop and utilize computer programs generated by or available to the ASMAR Program in support of Headquarters and Jet Propulsion Laboratory efforts to explore the usefulness of solar-electric propulsion for interplanetary-planetary and other advanced missions. This work is being carried out by the ASMAR Program under NASA Contract No. NSR 31-001-078 and is reported separately.

The support provided the ASMAR Program by personnel of the Nuclear Propulsion Research Laboratory of Guggenheim Laboratories has been compromised by uncertainties in the future status of that Laboratory. In the meantime initial efforts to establish an approach to aerospace systems analysis, including nuclear propulsion systems, by personnel of

^{*}This research is supported by the NASA Office of Space Science and Applications Launch Vehicle and Propulsion Programs Division under Contract NAST-231. Mr. J. W. Haughey is the NASA Program Monitor.

the ASMAR Group continue, but will come to later fruition than had been planned.

B. Personnel

A list of personnel who were active in the ASMAR Group as of 1 October 1966, the beginning of a new Program Year, is provided as APPENDIX B.

With respect to personnel activity, a paper by Dr. Morris
Handelsman (1) who continues as a consultant, was presented at the
International Astronautical Congress in Madrid in October. A paper
entitled "The Primer Vector on Fixed Time Impulsive Trajectories,"
co-authored by Dr. P. M. Lion and Dr. Handelsman (2) has been accepted
for presentation at the AIAA 5th Aerospace Sciences Meeting in New York
in January 1967.

Dr. Lion has been promoted to Assistant Professor and will continue to supervise the trajectory and guidance analysis efforts of the Group.

Dr. Colin N. Gordon of Radio Corporation of America has joined the Group and as of October 1 is on a half-time basis. He is developing optimization programs for solar electric powered spacecraft trajectories.

Mr. John H. Campbell of Analytical Mechanics Associates joined as of December 1 on a full-time basis under the subcontract with AMA.

Mr. Campbell's main responsibility will be the development of the ITEM n-body integration program.

Mr. Leonard B. Jas also joined the Group on December 1 as a Junior Draftsman.

The current list of graduate students includes Mr. George A.

Hazelrigg, Assistant in Research who was admitted in 1963. Mr. Hazelrigg has passed the general examination for the Ph.D. and is now on leave to JPL for six months in connection with the Solar Electric Mission Studies. He will return in February 1967 to complete his dissertation on powered spaceflight in the presence of two gravitating bodies including the problem of optimal escape and capture. Mr. Sol M. Rocklin, Assistant in Research who was admitted in 1964 will complete his MSE degree requirements by June 1967. His thesis "A Study of Similarities in the Primer Vector." Mr. Jean P. Peltier, Assistant in Research, was admitted in 1965, and Mr. Michael Minkoff, NDEA Fellow, was admitted in 1966. They are working on optimum impulsive trajectories (see II below).

Three undergraduates of the Class of 1967 are working on Senior Theses within the ASMAR Program. Mr. R. A. Philips is working on transfers with fixed thrust angles. Mr. M. J. Flynn is completing work on nuclear rocket systems. Mr. J. E. Kerr is considering the differences between direct and indirect ascent of launch vehicles for interplanetary missions.

Mr. E. J. Sarton of the Class of 1968 has initiated a study of solar activity.

C. Princeton University Computer Center

The Computer Center provides strong support in its capability for satisfying the Program's need for rather extensive computations.

Charges are covered in the University's indirect expenses. The University has been notified that it will be required to institute direct charging for all computing unless it can demonstrate a clear advantage to the Government by retention of the present or some alternative method.

Direct charging could have a great effect on the ASMAR Program so these developments are being closely watched.

D. Publications

A List of Publications and a List of MARS Memos, our internal publications, are given in APPENDICES D and E respectively as of the end of this reporting period. Copies of these publications are available on request from Miss Frances Allison, Engineering Quadrangle Room C-404, Princeton University, Princeton, New Jersey 08540.

II. SPACEFLIGHT TRAJECTORY ANALYSIS RESEARCH*

The capabilities of the group in spaceflight trajectory analysis research have increased significantly over the past nine months, the result of three years' development. Contributions have been and are currently being made in both the theoretical and numerical aspects of this field.

The theoretical work is concentrated in the area of optimum impulsive trajectories. This is consistent with the idea of extending the impulsive iterative method, originally developed here, to cover all possible combinations of boundary conditions. In brief, the technical contributions are:

- 1. Extension of the concept of the primer vector to non-optimum trajectories.
- 2. Proof of necessary conditions for optimal initial and final coasting.
- 3. Proof of the "criterion for an additional impulse." This tells when a given trajectory can be improved by a mid-course burn.
- 4. Proof of the transversality condition for non-optimal impulsive trajectories. This condition tells how the mid-course burns at a given trajectory should be changed to improve the trajectory.

For further details consult References (1) and (2). These theoretical results are presently being programmed by Messrs. Minkoff and Peltier. Using the third and fourth results above, it is possible to devise a gradient scheme which will determine the optimum n-impulse

^{*}Mr. William E. Miner, NASA Electronics Research Center, is the Technical Monitor of this phase of the research.

trajectory from the two-impulse or Lambert solution. It is expected that this program will be working by June 1967. It will be a unique capability (i.e., generation of the optimum impulsive trajectory) and will provide a solution to an area which has been almost completely ignored.

Mr. Sol Rocklin is writing a Master's thesis on the extension of the primer vector idea to certain singular cases.

These impulsive trajectories have inherent interest of themselves since they provide an excellent approximation for chemical rockets. However a second use of this program will be to generate adjoint vectors for finite thrust trajectories in the impulsive-iterative technique.

It is planned that the program being developed by Mr. M. Minkoff will be coupled with that developed by Professor Lion to provide a complete impulsive iterative capability. Professor Lion's program generates optimum finite (fixed) thrust trajectories. It is written in cartesian coordinates and represents the following advances over our previous programs:

- 1. three dimensions
- analytical coasting arcs
- 3. analytical partial derivatives

This program is presently converging. Trajectories on the 7094 are computed generally in 2-7 seconds of machine time.

The program by Dr. C. N. Gordon, although developed principally under the solar electric contract, has been the bellwether of our efforts this year, while Dr. Lion's program was being written. It uses analytical partial derivatives and has the capability of computing solar electric

powered thrustors as well as fixed thrust. It has demonstrated a truly amazing radius of convergence (generally up to 1 A.U., far greater than anything reported in the literature). This, we believe, illustrates the value of analytically computed derivatives. The program is written in two dimensions and hence will eventually be replaced in our work by the three dimensional program. It represents a powerful tool and is now being put into final (user) form by Dr. Gordon.

The body of work just described represents the heliocentric portion of the mission only. A significant contribution both theoretical and numerical has been made. During the coming year it is expected that out attention will concentrate more heavily on patching in the planetocentric phases. Mr. George A. Hazelrigg, during his stay at JPL has intensively researched this aspect of the problem. The proposed topic of his Ph.D. dissertation is the application of two-variable expansion techniques to problems with two gravitational centers.

Included within the work performed by Analytical Mechanics
Associates, the FORTRAN IV version of the ITEM program was checked for self-consistency and for integrator accuracy in the following manner.

1. A trajectory was integrated toward Mars for 230 days and 15 hours. Then using these terminal conditions as initial conditions, the trajectory was integrated backwards to time equal zero. Since the program uses a modified Encke scheme, the forward and backward sweeps are done differently and actually use slightly different conic sections. This test was done both with the solar electric propulsion on and off (i.e., coasting) with very accurate results.

2. An attempt to integrate the path of the planet Venus was made. The perturbations due to the planet Mercury were not included in these calculations. The distance of the integrated points from the expected positions of Venus according to a JPL ephemeris were printed out. Quite precise results were obtained with a distance from the "true" Venus position of 371.44639 km after 474 days.

III. AEROSPACE SYSTEMS ANALYSIS RESEARCH

Although reduced in pace as described above, activity continues in the topic area of Aerospace Systems Analysis.

Dr. Robert Vichnevetsky of Electronic Associates, Inc., who is a part-time Visiting Research Scientist, has continued his seminars on computational methods for aerospace systems analysis. The use of computers in the solution of partial differential equations is of particular importance in thermodynamic analyses and in the selection of optimal sets of parameters for propulsion systems.

Work has continued on the optimization analysis of nuclear rocket systems based on the efforts of Mr. M. J. Flynn (3). Studies are also underway on the nuclear electric Brayton cycle space power systems as an extension of a previous MSE thesis (4).

Studies of space electric rocket propulsion systems are also being continued.

Other systems, such as payload systems, communications systems and controls, guidance and navigation systems, are being considered for study and analysis under the ASMAR Program and these will be discussed in a later report.

IV. INTERPLANETARY-PLANETARY MISSION ANALYSIS RESEARCH

Little work has been carried out in this topic area in the Basic Program during the period of this report because all of our rather limited capabilities have been concentrated on initiation of the Solar Electric Mission studies under NASA Contract No. NSR 31-001-078. This was understood when the solar electric work was undertaken; however, we are attempting to formulate the computer programs for that work and design their capabilities so they can use constant or other power profiles (e.g., as in the nuclear electric case) for low thrust and "high" thrust as well. The need for analyzing missions optimally on a three-dimensional basis including both heliocentric and geocentric phases using actual planetary ephemerides is being kept in mind.

APPENDIX A: References

- Handelsman, M., Some Necessary Conditions for Optimal Fixed-Time Powered Transfers with Multiple Coasts and Thrusts Between Circular Orbits, XVII International Astronautical Congress, Madrid, Spain, October 13, 1966. (MARS Memo #27)
- 2. Lion, P. M. and Handelsman, M., The Primer Vector on Fixed-Time Impulsive Trajectories, (AIAA Paper No. 67-54). Also Princeton University AMS Report No. 717. (In Preparation)
- 3. Flynn, M. J., Performance Analysis of the Solid Core Nuclear Rocket 1975-85, Princeton University AMS Report No. 717k (Limited Distribution), 30 December 1966.
- 4. Burton, C. D. and Evans, J. A., Nuclear Space Power System Analyses for the Unmanned Mars Round Trip, Princeton University AMS Report No. 717d (Limited Distribution), 20 May 1965.

PRINCETON UNIVERSITY Department of Aerospace and Mechanical Sciences

As of 1 October 1966

AEROSPACE SYSTEMS AND MISSION ANALYSIS RESEARCH (ASMAR) PROGRAM

Personnel List

<u>Administrative</u>	J. P. Layton, Research Leader F. Allison, Senior Project Secretary (, Project Secretary	pt ft ft)		
Spaceflight Trajectory Analysis Research		1/2t ft leave) 1/2t 1/2t 1/2t,nc pt		
Aerospace Systems Analysis Research	J. P. Layton, Senior Research Engineer R. Vichnevetsky, Visiting Research Scientist P. M. Williams, Research Staff Member M. J. Flynn, Undergraduate Student '67	1/4t 1/5t pt pt		
Planetary-Interplanetary Mission Analysis Research	J. P. Layton, Senior Research Engineer A. B. Shulzycki, Programmer J. S. Wood, Grad Fellow (MSE Cand) J. E. Kerr, Undergraduate Student '67 E. J. Sarton, Undergraduate Student '68	1/4t ft 1/2t,ne pt pt		
Consul	ltants: L. Crocco, Professor	n+ na		
	D. Graham, Professor	pt,nc 1/6t		
	J. Grey, Associate Professor R. G. Jahn, Associate Professor R. A. Phinney, Associate Professor	pt,nc pt,nc pt,nc		
	M. Handelsman, Professor (Drexel)	1/5t		
	A. E. Bryson, Professor (Harvard/MIT)	pt		
	G. Leitmann, Professor (U.of Cal., Berkeley)	pt		
Subcontracts:				
	AMA - S. Pines, H. Kelley, et al.	pt		
	RCA - C. Gordon	1/2t		

APPENDIX C: Princeton University Computer Center Capability and Planning, As of 1 April 1966

Professor Edward J. McCluskey, who is the director of the Computer Center, will be replaced by Mr. Roald Buhler, currently an assistant director, as of 1 July 1966. Mr. Hale F. Trotter is the associate director and programming manager; Mr. Theodore A. Dolotta is an assistant director; Mr. Edward G. Aubin, Jr., assistant to the director; Messrs. R. Baumberg, A. M. Jones, Jr. and L. Young are programming staff members; and Mr. A. B. Adams is operations manager.

The Princeton University Computer Center comprises all of the stored-program computer installations on campus. The major installation, which is located in the Engineering Quadrangle, includes an IBM 1410 computer, an IBM 7094 computer with an associated cathode ray tube display system, and an IBM 7044-1401 computer system. At the Forrestal Research Campus there is an IBM 1410 computer system at the Plasma Physics Laboratory, and IBM 360 model 40 computer at the Princeton-Pennsylvania Accelerator, and an IBM 1620 computer in the Guggenheim Laboratories for the Aerospace Propulsion Sciences. In addition, the Computer Center administers twenty hours per week of University use of a CDC 1604 computer owned by the Institute for Defense Analyses and housed in von Neumann Hall adjoining the campus.

An IBM system 360/50 will be installed on main campus in the latter half of 1966. This 360/50 will be replaced by an IBM 360/67 twin processor system in the summer of 1967 at which time the Computer Center will occupy a new building.

All of the computers are available only to University students and staff. No charges are ever made for computer time. There are opportunities for any student or staff member to operate each of the computers himself, and special operators are normally provided only at the Engineering Quadrangle installation. The Computer Center staff provides training seminars on the use of specific programming languages and a daily, clinic is conducted by a staff member to provide individual help for specific problems. It is the responsibility of the problem originator to carry out the detailed preparation of his program.

APPENDIX D:

PRINCETON UNIVERSITY Department of Aerospace and Mechanical Sciences

AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH (ASMAR) PROGRAM

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List of Publications

Constantine, R. W., An Analysis of a Ramjet Propelled Recoverable Launch Vehicle State, (Senior Thesis - June 1964), Princeton University AMS Report No. 717a (Limited Distribution, 3 June 1964.

Richardson, W. P., Investigations of a Hybrid Rocket Powered Unmanned Mars Excursion Vehicle, (Senior Thesis - June 1964), Princeton University AMS Report No. 717b (Limited Distribution), 3 June 1964.

Handelsman, M., Hazelrigg, G. A., Hoffman, L. L. and Wallack, P. J., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 1, Overall Presentation, Princeton University AMS Report No. 717c-1, 29 January 1965.

Wallack, P. J., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 2, Computer Program, Princeton University AMS Report No. 717c-2, 29 January 1965.

Hoffman, L. L., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 3, Tabulated Results for Earth-to-Mars Transfer, Princeton University AMS Report No. 717c-3, 29 January 1965.

Burton, C. D. and Evans, J. A., Nuclear Space Power System Analyses for the Unmanned Mars Round Trip, (Joint MSE Thesis - May 1965), Princeton University AMS Report No. 717d (Limited Distribution), 20 May 1965.

Handelsman, M., Optimal Free-Space Fixed-Thrust Trajectories Using Impulsive Trajectories as Starting Iteratives - Volume 1, Theory, Princeton University AMS Report No. 717e-1, 31 March 1966.

Wallack, P. J., Optimal Free-Space Fixed-Thrust Trajectories Using Impulsive Trajectories as Starting Iteratives - Volume 2, Computer Program, Princeton University AMS Report No. 717e-2, (In Preparation).

Vichnevetsky, R., Lotito, L., Jones, K., Hoffman, L. L., Preliminary Results in the Development of a Hybrid Digital Trajectory Optimization Method, Princeton University AMA Report No. 717f (Limited Distribution), 7 June 1966.

Hanks, T., The Meaning of a Heat Flow Experiment on Mars (Senior Thesis - April 1966), Princeton University AMS Report No. 717g (Limited Distribution), 7 June 1966.

Hansmann, D. R., Optimization of Three-Burn, Double-Conic, Heliocentric Fixed Time Trajectories in Two Dimensions by Use of the Method of Parallel Tangents (Senior Thesis - June 1966), Princeton University AMS Report No. 717h (Limited Distribution), 7 June 1966.

Armstrong, T., TRSYS - A Trajectory and Mission Analysis System Program (Senior Thesis - June 1966), Princeton University AMS Report No. 717i, (Limited Distribution), 7 June 1966.

Handelsman, M., Some Necessary Conditions for Optimal Fixed-Time Powered Transfers with Multiple Coasts and Thrusts Between Circular Orbits, Princeton University AMS Report No. 717j, 13 October 1966.

Flynn, M. J., Performance Analysis of the Solid Core Nuclear Rocket 1975-1985 (Junior Independent Work - September 1966), Princeton University AMS Report No. 717k (Limited Distribution), 30 December 1966.

Lion, P. M. and Handelsman, M., Primer Vector on Fixed-Time Impulsive Trajectories, Princeton University AMS Report No. 7172, (In Preparation)

PRINCETON UNIVERSITY Department of Aerospace and Mechanical Sciences

AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH (ASMAR) PROGRAM

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List of MARS Memos

MARS Memo #1, Significance of Lagrangian Multipliers, Earth-Mars Circular Orbit Transfer, M. Handelsman, February 1964.

MARS Memo #2, Derivative of Switching Function K(t) Earth-Mars Circular Orbit Transfer, M. Handelsman, February 1964.

MARS Memo #3, Useful Form of Weierstrass Necessary Condition for Mayer Type Problem, M. Handelsman, February 1964.

MARS Memo #4, Use of Weierstrass Necessary Condition Earth-Mars Circular Orbit Transfer, M. Handelsman, February 1964.

MARS Memo #5, Some Notes on Talk by Prof. A. E. Bryson at Princeton University of 13 February 1964 on Control Optimization, M. Handelsman, February 1964.

MARS Memo #6, Current Research Earth-Mars Round-Trip Trajectory Optimization Using Low-Thrust Propulsion, M. Handelsman, 26 February 1964.

MARS Memo #7, Analytic Solutions to Variational Euler Equations During Coast, M. Handelsman, 6 March 1964.

MARS Memo #8, Suggestion for Checking an Iteration Program, M. Handelsman, 12 March 1964.

MARS Memo #9, Variational Equations for Optimized Constant Thrust One-Way Transfer from Earth (Circular Orbit) to Mars (Elliptic Orbit), Coplanar Case, M. Handelsman, 13 March 1964.

MARS Memo #10, Some Objectives in Trajectory Analysis of Earth-Mars Round Trips with Provision for Mars Excursion, M. Handelsman, 16 March 1964.

MARS Memo #11, Progress on Optimum One-Way, Circular-Orbit Transfer Earth-Mars, M. Handelsman, 25 March 1964.

MARS Memo #12, Differential Equations for the Solution of the Partial Derivative Matrix Required for a Newton-Raphson Iteration Scheme, G. Hazelrigg, 8 May 1964.

MARS Memo #13, Computation of Planetary Ephemerides, G. A. Hazelrigg, 20 May 1964.

MARS Memo #14, State of the Art Review, G. A. Hazelrigg, 4 June 1964.

MARS Memo #15, Optimization of Interplanetary Round Trips Using Low Thrust Propulsion - I, M. Handelsman, 8 June 1964.

MARS Memo #16, Normalization of Optimized Trajectory Equations for Interplanetary Flight, M. Handelsman, 16 November 1964.

MARS Memo #17, The Behavior of the Switching Function and Propulsion Requirements for Trajectories Optimized for Minimum Flight Time of Minimum Flight Angle, M. Handelsman, 25 November 1964.

MARS Memo #18, Calculation of Minimum-Fuel Optimal-Time Two-Impulse Transfers, M. Handelsman, 18 March 1965.

MARS Memo #19, Fixed-Time, Two-Impulse, Interplanetary Transfers Using a Sphere-of-Influence Concept, G. A. Hazelrigg, 8 June 1965.

MARS Memo #20, Investigation of Double Minima in the Impulse Function of a Two-Impulse, Fixed Angle Orbital Transfer, S. M. Rocklin, 23 August 1965.

MARS Memo #21a, Techniques of Analyzing Planetary Escape and Capture Trajectories and Patching on to Heliocentric Transfers, Part I, G. A. Hazelrigg, 1 March 1966.

MARS Memo #21b, Techniques of Analyzing Planetary Escape and Capture Trajectories and Patching on to Heliocentric Transfers, Part II, S. M. Rocklin, (In preparation).

MARS Memo #22, Solution of the Equations of Low-Thrust Escape and Capture Via the Edelbaum Asymptotic Matching Technique, G. A. Hazelrigg, 28 February 1966.

MARS Memo #23, Record of Meeting on the Solar-Electric Propelled Spacecraft Mission Analysis, G. A. Hazelrigg and S. M. Rocklin, 9 March 1966.

MARS Memo #24, Preliminary Numerical Tests of the Convergence Power of a Newton-Raphson Iteration Technique, M. Handelsman, 6 April 1966.

MARS Memo #25, Preliminary Results on Some Necessary Conditions for Optimal Fixed-Time Transfers with Multiple Coasts and Burns Between Circular Orbits, M. Handelsman, 9 May 1966.

MARS Memo #26, Extensions of the Theory of Impulsive, Two Burn Fixed-Time, Fixed-Angle Trajectories Between Circular Given Orbits, J. P. Peltier, (In preparation).

MARS Memo #27, Some Necessary Conditions for Optimal Fixed-Time Powered Transfers with Miltiple Coasts and Thrusts Between Circular Orbits, M. Handelsman, 13 October 1966. (Revised MARS Memo #25)